Hypothesis Testing

Problem 1: A car manufacturer claims that their vehicles' fuel efficiency is at least 5 MPG better than a competitor's vehicles. The competitor's vehicles have an average fuel efficiency of 25 MPG. A sample of 20 cars from the manufacturer shows an average fuel efficiency of 31 MPG with a standard deviation of 4 MPG. Test the claim at the 0.05 significance level.

Problem 2: A fertilizer company claims that their product increases crop yield by more than 20%. The average crop yield without the fertilizer is 400 kg per hectare. A sample of 10 farms using the fertilizer shows an average yield of 480 kg per hectare with a population standard deviation of 30 kg. Test the claim at the 0.01 significance level.

Chi-Square Testing for Variances

Problem 4: Test the claim H0: σ 2=4 vs. Ha: σ 2>4 at the 0.05 significance level, given a sample of 25 observations with a mean of 100 and a standard deviation of 2.5.

Problem 5: A researcher claims that the variability in response times for a new software system is lower than 3 seconds. A sample of 18 trials yields a standard deviation of 2.2 seconds. Test the claim H0: σ =3 vs. Ha: σ <3 at the 0.01 level.

Proportions Testing

Problem 6: A city council claims that more than 60% of its residents support a new recycling initiative. Out of a random sample of 1,200 residents, 750 say they support the initiative. Perform the test at the 0.05 significance level.

Problem 7: Test the claim H0:p^=0.4 vs. Ha:p^>=0.4, given that out of 500 people surveyed, 210 exhibits a certain preference. Use a 0.01 significance level.

Type I and Type II Errors

Problem 9: A sample of 20 observations is collected to test H0: μ =100 vs. Ha: μ >100 at the 0.01 significance level. The population standard deviation is known to be 5. The sample mean is 102. Calculate the p-value and determine if an error is made if μ =99.

Problem 10: A sample of 15 data points is collected to test H0: μ =75 vs. Ha: μ <75 at the 0.05 level. The population standard deviation is 2.8. The calculated test statistic is -1.95. Determine the p-value and evaluate the test outcome if μ =73.5.

Two-Sample Analysis

Problem 11: Test the claim that two new teaching methods produce the same average student test scores at the 0.05 level. Data:

- Method 1: n=20, x⁻=75, s=8n=20, x⁻=75,s=8
- Method 2: n=18, x⁻=72, s=6n=18, x⁻=72,s=6

Problem 12: Test the claim that the average lifespan of batteries from two manufacturers differs at the 0.01 level. Data:

- Manufacturer A: n=12, x⁻=500, s=25n=12, x⁻=500,s=25
- Manufacturer B: n=15, x⁻=510, s=20n=15, x⁻=510,s=20